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LETTER TO THE EDITOR

Scaling of level statistics and critical exponent of disordered two-dimensional symplectic systems

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Abstract. The statistics of the energy eigenvalues at the metal–insulator transition of a two-dimensional disordered system with spin–orbit interaction is investigated numerically. The critical exponent ν is obtained from the finite-size scaling of the number J_0 which is related to the probability $Q_n(s)$ of having n energy levels within an interval of width s. In contrast to previous estimates, we find $\nu = 2.32 \pm 0.14$ close to the value of the two-dimensional quantum Hall system.

Despite considerable efforts the understanding of the critical behaviour at the disorder driven metal–insulator transition (MIT, Anderson transition) in disordered systems is still incomplete. Present day analytical theories are not able to provide quantitative results of physical quantities which describe the universal properties in the vicinity of the critical point. Therefore, our current knowledge mainly originates from numerical investigations using transfer-matrix approaches and Green function techniques (see e.g. [1]). Recently, the energy level statistics has proven to be another powerful method to elucidate the peculiar spectral correlations and to calculate the non-trivial exponents that govern the non-conventional dynamics [2, 3, 4, 5] that are related to the multifractal properties of the corresponding eigenstates.

In particular, the energy level spacing distribution $P(s)$ is a simple tool to distinguish between localized and extended states, and also reflects the respective symmetry of the model system under consideration. Here, $s = |E - E'|/\Delta$ is the energy separation of two consecutive eigenvalues E and E' divided by the mean level spacing Δ. On the metallic side of the MIT, random-matrix theory (RMT) serves as an adequate description. However, in approaching the transition, novel critical level statistics have been found in three-dimensional (3d) [6, 7, 8, 9, 10] and two-dimensional (2d) [11, 12, 13, 14] systems. For small level separations, s, these critical $P_c(s)$ still resemble the RMT result, $P(s) \sim s^\beta$, indicating strong level repulsion, while for large spacings a behaviour, $P(s) \sim \exp(-\kappa s)$, similar to the uncorrelated Poisson statistics of the localized states is observed. Previously, we have found the size-invariant $P_c(s)$ for 2d symplectic [11] and QHE systems [14] with $\kappa \approx 4.0$, and for 3d orthogonal and unitary models [9] where $\kappa \approx 1.9$ was obtained.

A disorder driven metal–insulator transition is observed in 3d for each possible symmetry class: orthogonal ($\beta = 1$), unitary ($\beta = 2$) and symplectic ($\beta = 4$). In non-interacting 2d systems, however, in the presence of both time-reversal and spin-rotational symmetry (orthogonal) all states are believed to be localized for any disorder $W > 0$. The same holds also in the absence of time-reversal symmetry (unitary), but singular energies are present in

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strong magnetic fields where the localization length diverges leading to the quantum Hall effect.

The only complete MIT in 2d is found for symplectic systems that possess time-reversal but no spin-rotational symmetry. Therefore, the symplectic model is of particular interest for developing analytical theories and the knowledge of characteristic physical quantities is of importance. Yet, there is no consensus about the magnitude of the universal critical exponent ν that governs e.g. the divergence of the localization length, $\xi(E) \sim |E - E_0|^{-\nu}$, at the MIT, whereas the value $\nu \approx 2.35$ for the QHE case is commonly accepted [15, 16, 17]. The proposed values for the symplectic system have been found by using the transfer matrix method. They are scattered over a wide range: $\nu = 2.05 \pm 0.08$ at $W_c/V = 5.875 \pm 0.010$ [18, 19], $\nu = 2.75 \pm 0.15$ at $W_c/V = 5.74 \pm 0.03$ [20, 21], and $\nu = 2.5 \pm 0.3$ for a different model [12]. All suggested ν-values are clearly different from the QHE case which is contrary to the observed closeness of the generalized fractal dimensions $D(q)$ in symplectic and QHE systems [22]. Since several recent investigations (e.g. [11, 22, 23, 24]) at the MIT were performed taking the numbers ($\nu = 2.75$ and critical disorder $W_c/V = 5.74$) published by Fastenrath [20, 21] for granted, an independent check of the critical exponent and disorder is necessary.

Three different models [18, 25, 26] have been proposed for numerical studies of the localization properties in 2d disordered systems with symplectic symmetry. In the present investigation we use the model suggested by Ando in order to be able to compare with the above mentioned results for the critical exponent. Also, this model seems to be more realistic, because it simulates transfer of electrons between s-orbitals via p-orbitals in the presence of spin–orbit interaction. The latter is responsible for the symplectic symmetry due to the broken spin-rotational invariance. In the second quantization, the Hamilton operator on a square lattice with sites m and n and lattice constant a is

$$H = \sum_{m,\sigma} \varepsilon_m c^{\dagger}_{m,\sigma} c_{m,\sigma} + \sum_{\langle m \neq n \rangle, \sigma\sigma'} V(m, \sigma; n, \sigma')\, c^{\dagger}_{m,\sigma} c_{n,\sigma'} \tag{1}$$

where the disorder potentials ε_m are random numbers distributed between $-W/2$ and $W/2$ with probability $P(\varepsilon) = V/W$. Periodic boundary conditions are applied in both directions. The spin–orbit interaction strength S is defined as the ratio $S = V_2/(V_1^2 + V_2^2)^{1/2}$, where V_1 and V_2 are matrix elements of the 2×2 complex transition matrices $V(m, \sigma; n, \sigma')$, which depend on the transfer direction and on spin σ. In the following we choose $S = 0.5$ and the unit of energy $V \equiv (V_1^2 + V_2^2)^{1/2} = 1$, and only nearest-neighbour transfer is considered. The twofold degenerate eigenstates are calculated by direct diagonalization using a Lanczos algorithm.

The eigenvalue statistics are calculated in a given interval $[-0.5\,V,\ 0.5\,V]$ around the band centre $E/V = 0.0$. A careful spectral unfolding procedure was applied to compensate for possible global variations in the density of states which would disturb the interesting local correlations. More than 10^6 eigenvalues were accumulated for each of the parameter pairs (L, W) with linear system size $L/a = 15, 20, 25, 30, 35, 40, 100$ and disorder strength ranging from $W/V = 5.4$ to 6.8 by calculating up to 3000 realizations of the disorder potentials. As a scaling variable, we consider the quantity [27]

$$J_0(L, W) = \int_0^{\infty} Q_0(s)\, \mathrm{d}s \tag{2}$$

where $Q_n(s)$ for $n = 0$ is the probability that an energy interval of width s contains no energy eigenvalue. It is related to the nearest-neighbour level spacing distribution, $P(s) \equiv \mathrm{d}^2 Q_0(s)/\mathrm{d}s^2$ [28]. Compared to previous approaches [3, 6, 7] the major advantage

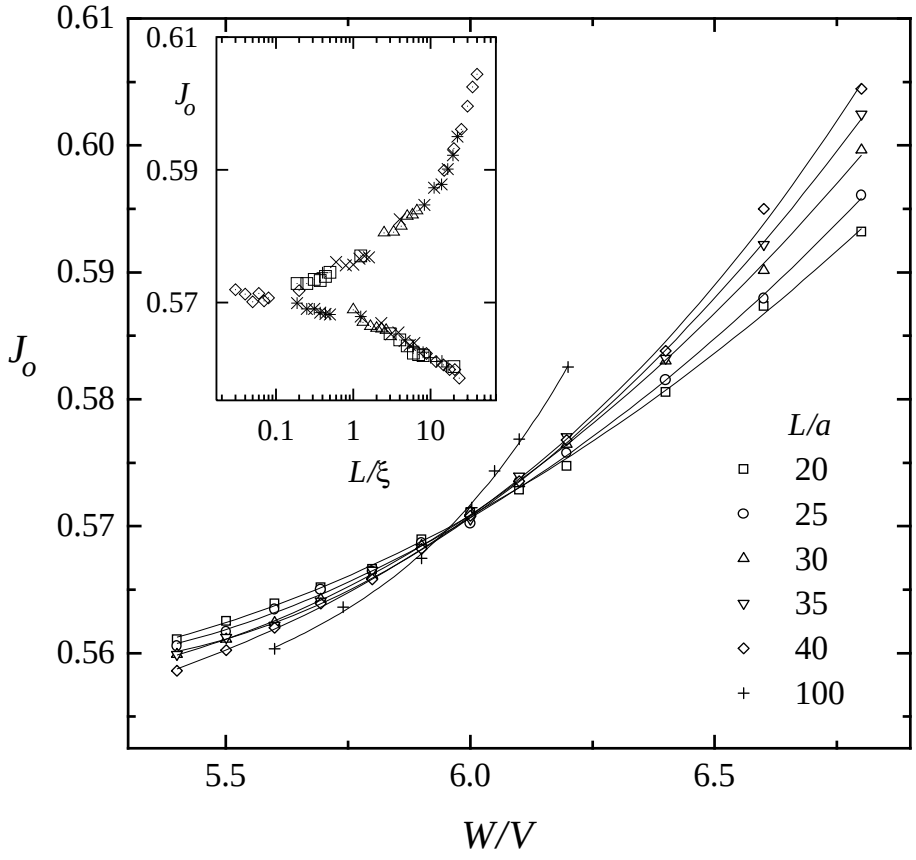

Figure 1. The scaling variable J_0 versus the disorder W for various linear size L of the 2d square lattice. The inset: the two-branch scaling curve $J_0(L/\xi)$ showing the metal–insulator transition at $J_0^c \approx 0.571$. Different symbols correspond to different disorder.

of our choice is that all calculated neighbouring level spacings are used, because no arbitrary cut-off parameter is needed.

The spectral correlations of a disordered system undergo a crossover from the Wigner to the Poisson statistics when increasing the disorder [6, 7, 8]. In the metallic phase, $J_0 \approx 0.522$ is well known for infinite symplectic systems from RMT [28], and in the insulating phase from the Poisson distribution $J_0 = 1$. As expected, the level statistic J_0 as a function of W changes from the RMT result ($W < W_c$) to the Poisson limit ($W > W_c$) continuously for finite L, but discontinuously in the thermodynamic limit. It exhibits critical behaviour close to the disorder W_c, which separates the extended and localized regimes. The sign of the finite-size effect is reversed when crossing the fixed point, signalling the delocalization–localization transition.

Results of $J_0(L, W)$ as a function of disorder are plotted in figure 1 for six different system sizes. A point of intersection at $W/V \approx 5.98$ is clearly observed. Within numerical error $\delta W \approx 0.04$, it is consistent with the value reported in [19], but certainly larger than that from [20]. We find that our $J_0(L, W)$ results fall between the limiting values mentioned above with a size independent $J_0^c = 0.571 \pm 0.001$ at the critical point. Similar behaviour is also found for $J_{n\geqslant 1}(L, W)$ by calculating the probability $Q_n(s)$ of the non-zero number

of eigenvalues n in the interval s. We obtained a scale-invariant sequence of the critical numbers J_n^c, which grow with n, $J_1^c \approx 0.976$, $J_2^c \approx 0.996, \ldots$, and converge very fast to unity. In 3d orthogonal systems, an analogous critical sequence [27] was found earlier.

Assuming the validity of the one-parameter scaling hypothesis, $J_0(L, W) = f(L/\xi(W))$, one can introduce the correlation length $\xi(W)$ and re-scale the linear size L so that the data of J_0 as a function of L/ξ for consecutive W overlap with each other. In approaching the critical point the precision of ξ decreases due to less overlap. Within statistical accuracy all points fall onto one common curve independently of L and W. This yields the two-branched scaling function displayed in the inset of figure 1, where $J_0(L, W)$ is plotted versus $L/\xi(W)$. The lower and upper branches indicate the extended and localized sides of a complete metal–insulator transition, respectively.

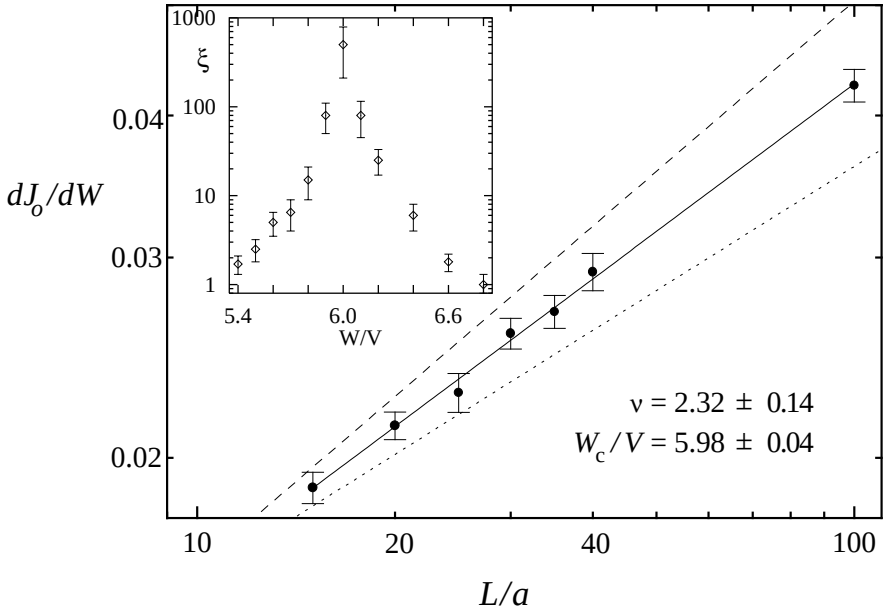

Figure 2. The derivative $\mathrm{d}J_0/\mathrm{d}W$ at W_c as a function of the system size L (log–log plot). The solid line is a linear fit to the raw data. Dashed and dotted lines correspond to the values of the critical exponent $\nu = 2.05$ and 2.75 obtained in [19] and [20], respectively. The inset shows the disorder dependence of the correlation length $\xi(W)$.

In order to calculate the critical exponent ν one can expand the finite-size scaling function near $W = W_c$,

$$J_0(L, W) \approx J_0^c + A(L/\xi(W))^{1/\nu} = J_0^c + A'(W - W_c)L^{1/\nu}. \tag{3}$$

This gives $\mathrm{d}J_0(L, W)/\mathrm{d}W \propto L^{1/\nu}$. As soon as W_c is detected one can perform the two-parameter linear fit on a double-log scale, the inverse slope being ν. Figure 2 shows the derivative $\mathrm{d}J_0(L, W)/\mathrm{d}W$ at $W_c/V = 5.98$ as a function of the system size L from which a critical exponent $\nu = 2.32 \pm 0.14$ can be extracted. Recent calculations of a symplectic network model yield a similar exponent [29]. As a guide to the eye, two additional lines are drawn for comparison, the slopes of which correspond to the values of ν obtained by Ando [18, 19] and Fastenrath [20, 21], using the transfer-matrix method.

If W_c is not known with sufficient accuracy, a four-parameter fitting procedure to equation (3) is required. By applying a χ^2-criterion similar to that from [30, 31], we

verified that the resulting W_c and ν do not change considerably. However, their uncertainties increase as expected, $\delta W = 0.10$ and $\delta\nu = 0.16$. Thus, our value of ν is clearly distinct from those obtained previously. Note that the obtained error bar $\delta\nu$ does not overlap with corresponding error bars of those ν from previous studies. The inset of figure 2 displays the correlation length ξ as a function of disorder W. Here, ξ is defined up to a constant factor ξ_0. It diverges at the critical disorder in agreement with the power law $\xi(W) = \xi_0|W - W_c|^{-\nu}$. It is generally believed that the value of ν is universal and does not depend on the strength S of the spin–orbit interaction. It remains to be checked whether this expectation really holds.

In conclusion, we have investigated the critical properties of the energy level statistics at the metal–insulator transition of a disordered 2d system with symplectic symmetry. The scaling function and the critical exponent were calculated using the statistics of spacings of neighbouring energy eigenvalues that have been obtained numerically. By performing a finite-size scaling analysis we found a critical exponent $\nu = 2.32$ at a critical disorder $W_c/V = 5.98$ which is markedly different from those values reported previously. Our value of ν for the symplectic system is very close to that found in 2d QHE systems, a behaviour that has been observed previously also for the generalized fractal dimensions.

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